Multivariate subdivision and a simple formula

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In the univariate case the smoothness of a refinable function ϕ, that is, a function that satisfies a *refinement equation*

$$\phi = \sum_{k \in \mathbb{Z}} a_k \, \phi \left(2 \cdot -k \right), \qquad a \in \ell_{00} \left(\mathbb{Z} \right),$$

can be characterized in terms of differences of the subdivision operator. In fact, one way to phrase this relationship is to say that

> a *stable* refinable function ϕ has a derivative of order k if there exists a mask b such that
> $$D^k S_a = 2^{-k} S_b D^k$$
> and S_b is a *convergent* subdivision scheme.

Here, the *subdivision operator* S_a is defined as

$$S_a \; : \; c \mapsto \left(\sum_{\alpha} a_{j-2k} \; : \; j \in \mathbb{Z} \right)$$

and D denotes the (sequence) difference operator

$$D \; : \; c \mapsto \left(c_{j+1} - c_j \; : \; j \in \mathbb{Z} \right).$$

The talk will consider a multivariate extension of this result and some of the interesting problems to be encountered on the way there.